

# Statistical Catch-At-Length assessment results for *Sebastes mentella* and *S. fasciatus* in Units 1 and 2

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## SUMMARY

*Past attempts at Statistical-Catch-At-Length assessments for the redfish populations in Units 1+2 have struggled to reconcile survey biomass trends with survey catch-at-length data. Here it is shown that reconciliation is possible under the assumptions of natural mortality decreasing with age, and a situation where only occasionally extraordinarily strong year classes enter the populations. While the species-disaggregated assessments developed here could be refined (and species-aggregated), it is suggested that first discussions should be held to agree or otherwise on the reasonableness/plausibility of these core aspects of the dynamics of these species. The situation of occasional extraordinarily strong year classes has implications, which are discussed, for the basis under which reference points for these populations are best evaluated.*

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## 1. Introduction

This document builds on the Statistical catch-at-length (SCAL) assessments for *S. mentella* and *S. fasciatus* in Units 1 and 2 which were reported in Rademeyer and Butterworth (2014). That document summarised that “Fitting the declines in the survey indices in Unit 1 for the earlier (pre 1995) years proves a particular problem .....allowing for occasional large recruitments in these populations shows promise for improving the fits to those survey indices. However this needs further investigation to determine whether associated poor fits to the survey catch-at-length data can be avoided.” The particular focus of the analyses that follow is towards resolving these problems within the framework of allowing for occasional large recruitments.

Initial attempts experimented, but without success, on the somewhat complex combined species assessment approach of Rademeyer and Butterworth (2014). Accordingly it was decided to simplify the problem, at least initially, to be better able to focus on the core “conflict” problems as indicated above. This involved carrying out species-disaggregated SCAL assessments, which consequently were unable to make use of the commercial catch-at-length data which are species aggregated. The focus was first on *S. mentella*. The line of assessment development which was followed can be summarised in the following steps.

- 1) First survey selectivity was adjusted to get a better fit to the survey catch-at-length (cal) data. A two-component (normal followed by logistic, with parameters specified from inspection of the trends in the year-averaged proportional cal data) form was used to be able to reflect the relative paucity of catches in the 20-30 cm range. However a better fit to the cal data still rendered the assessment unable to reproduce the downward trend in the survey unit 1 biomass index prior to 1995. The selectivity in the top two panels in Figure 1 - selectivity "as we think" - shows the form used by length followed by its corresponding at-age form.
- 2) The decrease in the survey biomass index pre-1995 is essentially a decrease in biomass of the fish from the 1981 peak (extraordinarily strong) recruitment, i.e. that were 9 years old in 1990. However for the growth curve input for *S. mentella* (Campana, pers. commn), these 9 years olds corresponded to the "hole" in the cal distribution centred at about 22 cm (see Figure 1 - left panel on the second row, which show the length distribution of fish of age 9). Attempts to modify the growth curve proved unsuccessful, however. The right panel on the second row of Figure 1 shows the quite inadequate resultant fit to the survey biomass index, where the individual colour blocks reflect the biomass contribution of each year class to each year's survey biomass index (and except for the most recent few years are all swamped by the contributions from the 1981 year class)..
- 3) Consideration of the NSW trajectories for the 1981 year class (Number/Selectivity/weight multiplicative combinations - bottom row, right panel of Figure 1) led to the realisation of the need for the lower ages in this year class to contribute more to the survey biomass. This was achieved by increasing  $M$  for the lower ages.
- 4) An idea of the magnitude of this higher  $M$  value was obtained by estimating the  $M$  necessary for the observed decrease in the survey estimates of biomass that would result if the biomass consisted only of a single year class (Figure 2). It transpired that given this higher value for  $M$  at younger ages, survey selectivity could be reasonably represented by the simple logistic form.
- 5) The value of  $M$  at older ages (previously 0.1 for *S. mentella*) was then decreased to get a better fit to the cal data at larger lengths – the assessment model otherwise predicted too few fish at these lengths as insufficient were surviving to be able to attain these lengths. This approach to improving the fit was preferred to increasing selectivity at larger lengths, given the attractive parsimony of the logistic form resulting under 3).
- 6) Figure 3 plots the natural mortality-at-age vector that resulted from this overall process. While further analyses could consider “smoothing” this vector, it was considered better to maintain greater simplicity at this stage of the assessment process.

In the interests of parsimony, the same  $M$  vector as developed above for *S. mentella* was used in the separate assessment of *S. fasciatus*.

## 2. Data and Methods

The data used for these analyses are listed in Appendix A.

The SCAL methodology is described in detail in Appendix B.

The proportion of biomass in unit 1 was initially assumed to be 70% of the total biomass for each species, based roughly on the relative sizes of the two areas. These proportions were then adjusted separately for each species so that the survey  $q$ 's would be less than 1 in each unit. The final proportions consequently assumed for unit 1 are 40% for *S. mentella* and 20% for *S. fasciatus* (see Table 1).

The commercial selectivities, initially based on results from a previous analysis (Rademeyer and Butterworth, 2014), were chosen to give reasonable fits to the species aggregated catch-at-length data (see Table 1 for the parameter values of the logistic forms assumed).

### 3. Results and Discussion

Summary results of the two separate assessments for *S. mentella* and for *S. fasciatus* are listed in Table 2. Note that estimates of the pre-exploitation spawning biomass  $K^{sp}$  given there are based on past recruitments omitting the few years with extraordinarily strong year classes.

Figure 4 plots the growth and estimated age-length distributions for each of the two species. Note that the fitting procedure selects a wider distribution for length-at-age (a larger  $\beta$  value – see equation B9) for *S. fasciatus* compared to *S. mentella*.

The spawning and total biomass trajectories assessed for each species are shown in Figure 5. Figure 6 plots the time-series of estimated stock-recruit residuals and recruitment for each species. Years with extraordinarily high recruitment are not included in the estimation of the stock-recruit relationship: 1961, 1973, 1981 and 2011 for *S. mentella*, and 1982 and 2011 for *S. fasciatus*. Both assessments reflect high recruitments in the starting year of 1960, particularly for *S. mentella*, but these estimates should not be viewed as particularly reliable. The reasons are first that the assumption of a starting unexploited equilibrium age-structure (equation B11), though difficult to avoid given the very limited data available to inform estimates for that time, will doubtless result in bias in these starting recruitment estimates. Secondly the only data that do inform on year class strengths for this period are the catches-at-length for the largest lengths sampled by the Hammond surveys of the mid-80s; the fact that the proportions of the Hammond catches at these lengths are rather large (see also discussion following on Figures 8 and 9) has a high influence on these 1960 recruitment estimates, which is the reason that an additional downweighting factor was applied to these Hammond catch-at-length data in the log likelihood (see text following equation B17).

The fits to the survey biomass indices are shown in Figure 7. These fits are relatively good for both species in that they do reflect the broad trends, both recently and particularly the declines from the mid-80s to the mid-90s. However for *S. fasciatus* the residual pattern is unusual over this last-mentioned period, being a reflection of a mismatch over 1989-1990 between the end of the Hammond series and the start of the Needler-Teleost series – a matter which perhaps suggests some discussion on the reliability of the assumption that these two series are comparable.

The input commercial selectivities and the estimated survey selectivities and fits to the commercial and survey catch-at-length data are plotted in Figures 8 and 9 for *S. mentella* and *S. fasciatus* respectively. (For the commercial data, the “fits” actually compare the predicted catch-at-length for the species concerned with the species-aggregated observed catch-at-length; these data were not included in the fit to the model, and the comparisons are shown simply as a consistency check.) Note the highly positive residuals at large lengths for the years of the Hammond data, which are downweighted in the log likelihood for the reason explained above. For proportions-at-length averaged over years, predictions fail to reflect the strong minimum in the distributions of the observations around 22 cm, particularly in the case of *S. fasciatus*. However, despite the admitted need to try to improve these fits, this broad “mis-fit” feature of the year-averaged comparison is perhaps not as serious as might normally be considered. The reason is that the occasional peak recruitments “imbalance” these year-averaged plots; viewed instead at the year class level in the bubble plots, discrepancies are not as extreme as the year-averaged plots might initially seem to suggest.

The “fits” to the species combined commercial cal data are shown in Figure 10, where the predicted cal have been computed by adding the results from the two separate species assessments. The agreements are reasonable,

though note that these are not true fits to the data as this information could not be included in the likelihood under the separate assessments framework. Hence these comparisons serve rather as a consistency check.

The Hessian-based confidence intervals for spawning and total biomass trajectories are shown in Figure 11, while the CVs on the estimated recruitments are given in Table 3. These CVs are quite large, and the projected upturn in total biomass for both species in recent years is indicated to be not that well determined. Nevertheless the CV on the strong 2011 *S. mentella* recruitment is much less than typical for other years, suggesting that there is nevertheless strong qualitative evidence of this large incoming year class.

#### 4. Concluding remarks

The major achievement of these analyses has been the demonstration that allowing for a decrease in natural mortality with age can lead to an assessment which is able to qualitatively reconcile the survey biomass index and catch-at-length data for both *S. mentella* and *S. fasciatus* in Units 1+2 – a bar which it would seem that any defensible assessment for these redfish would need to reach. This is achieved within an overall approach that is able to maintain a simple logistic form for survey selectivity-at-length, and emphasises allowance being made for occasional extraordinarily strong year classes, though the resultant estimates of biomasses and recruitments are not that precisely determined.

Before attempting to refine these assessments further though, it seems appropriate first to discuss and agree or otherwise on the reasonableness/plausibility of these core aspects now suggested for the dynamics of the redfish populations being considered (decreasing *M*-at-age and occasional extraordinarily strong recruitments). There may be other information that can legitimately be brought to bear regarding, for example, the form and magnitude of the natural mortality-at-age relationship, which could lead, *inter alia*, to changes in the function shown in Figure 3, which is based entirely on achieving a best fit to the data taken into account in the assessment presented here. While it is possible (and desirable) to now return to the joint species assessment approach of Rademeyer and Butterworth (2014), this too would seem to better await such a discussion.

A situation of occasional extraordinarily strong year classes also has important implications for the estimation of resource status and biomass reference points. Should these peak year classes be incorporated into these computations, or should they not (as assumed for the results for stock status reported in Table 2, and arguably constitutes a defensible approach as reference points should pertain to “normal” rather than to “exceptional” situations). This clearly has important management ramifications, as the status estimated for both species under this assumption (see Table 2) is well above any plausible limit reference point level.

#### Acknowledgements

We thank Daniel Duplisea and Bruce Atkinson for kindly advising on the updated data.

#### References

- McAllister M and Duplisea DE. 2012. Production model fitting and projection for Acadian redfish (*Sebastes fasciatus*) in Units 1 and 2. DFO Can. Sci. Advis. Sec. Res. Doc. 2012/103. iii + 34p.
- Punt, A.E. and Kennedy, R.B. 1997. Population modeling of Tasmanian rock lobster, *Jasus edwardsii*, resources. Mar. Freshw. Res. 48, 967–980.
- Rademeyer, R.A. and Butterworth, D.S. 2014. Statistical catch-at-length assessment of *S. mentella* and *S. fasciatus* in Units 1+2. Document presented to Canadian redfish assessment review meeting, Mont Joli, 9-10 April 2014: 31pp.

**Table 1:** Values specified on input to the assessment

	<i>S. mentella</i>	<i>S. fasciatus</i>
Steepness of the stock-recruit relationship $h$	0.98	0.98
$\sigma_R$	1.50	1.50
Natural mortality $M$ (see Fig. 3):		
Ages 0-13	0.40	0.40
Ages 14+	0.05	0.05
Proportion of the biomass in unit 1 ( $u^1$ ):	40%	20%
Commercial selectivity logistic parameters (eqn B21):		
Unit 1:		
$a$	1.2	1.9
$b$	26.2	29.2
Unit 2:		
$b$	1.2	2.3
$a$	20.9	27.6

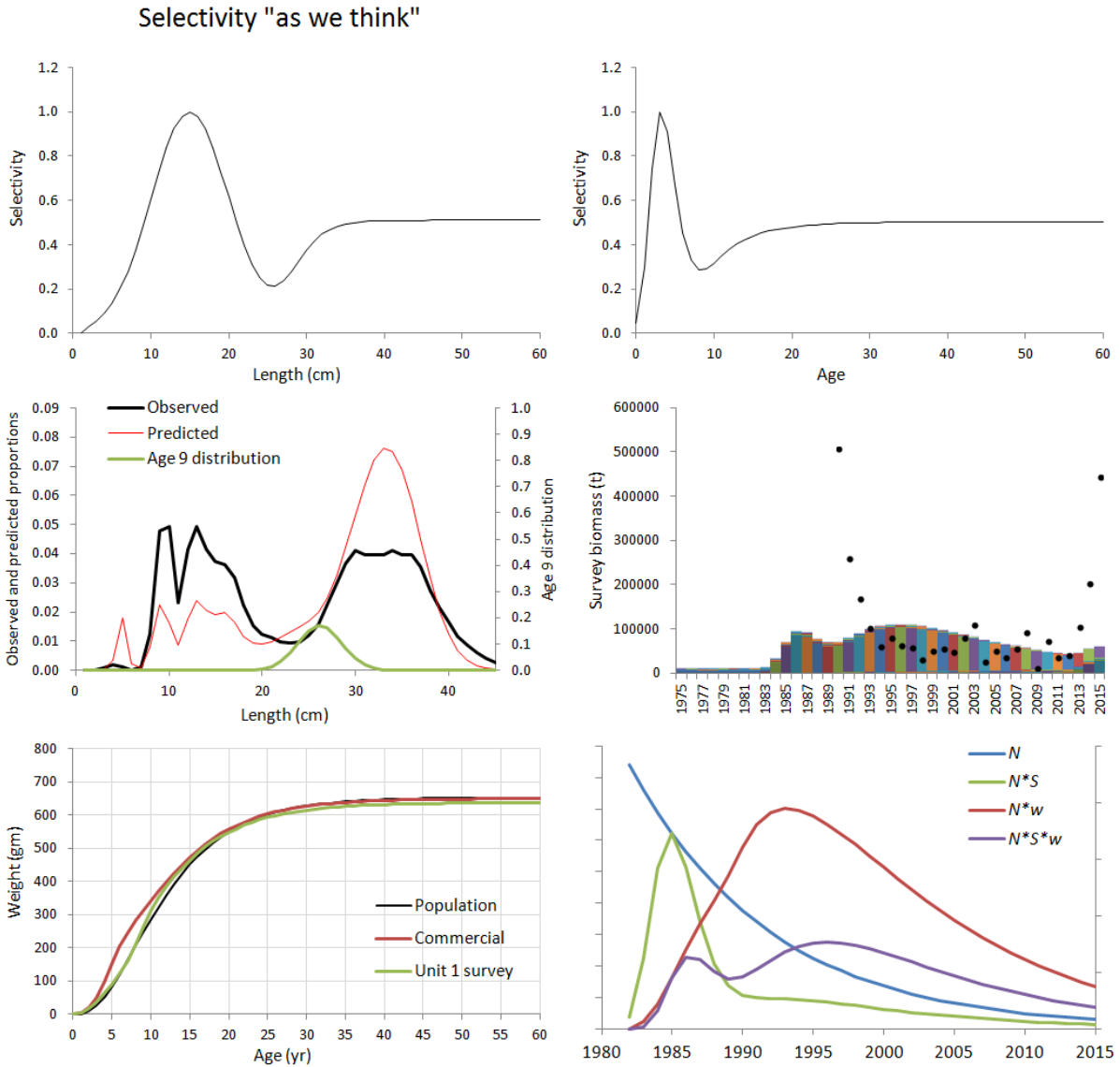
**Table 2:** Results for the SCAL Base Cases. Biomass units are in thousand mt, and  $K^{sp}$  refers to the pre-exploitation equilibrium spawning biomass. These results are based on the stock-recruit relationship omitting the few peak (extraordinarily strong) year classes (1961, 1973, 1981 and 2011 for *S. mentella* and 1982 and 2011 for *S. fasciatus*). The  $\sigma_{R\_out}$  are computed from the 1980+ recruitments, again excluding peak recruitment years.

	<i>S. mentella</i>	<i>S. fasciatus</i>		
Total -lnL	46.8	68.4		
-lnL <sup>catch</sup>	-76.2	-77.1		
-lnL <sup>survey</sup> unit 1	21.1	29.1		
unit 2	1.3	2.4		
caa_nll unit 1 surv	26.6	32.6		
unit 2 surv	1.4	6.6		
-lnL <sup>sr</sup>	72.6	74.9		
-lnL <sup>q</sup>	0.0	0.0		
$K^{sp}$	131.3	474.4		
$B^{sp}_{2015}$	173.1	196.3		
$B^{sp}_{2015}/K^{sp}$	1.32	0.41		
$\sigma_{R\_out}$	0.70	0.65		
Survey	Unit 1	Unit 2	Unit 1	Unit 2
$q$	0.56	0.97	0.91	0.96
$\sigma_{add}$	0.10*	0.10*	0.10*	0.10*

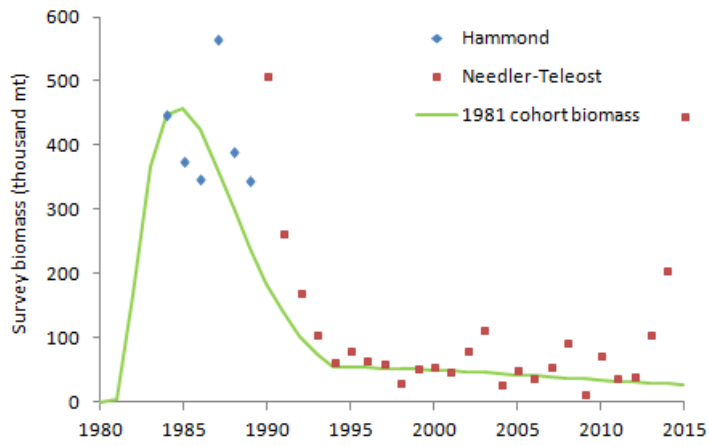
\* Hitting the lower bound imposed

**Table 3:** Values (in millions) and Hessian-based CVs for the estimated recruitments for *S. mentella* and *S. fasciatus*. Pre-1980, only the years with peak (extraordinarily strong) recruitments for *S. mentella* are shown (there are no such peaks estimated pre-1980 for *S. fasciatus*, and the data available are unable to reliably discriminate other recruitment variations for either species before about 1980).

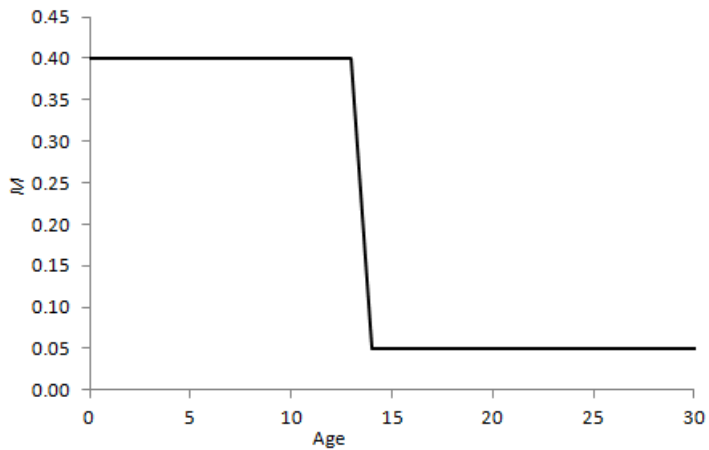
	<i>S. mentella</i>		<i>S. fasciatus</i>	
1961	396080	(0.33)	1551	(0.41)
1973	112570	(0.78)	1425	(1.48)
1980	628	(1.55)	1574	(1.60)
1981	121060	(0.39)	1441	(1.56)
1982	653	(1.56)	74651	(0.64)
1983	597	(1.51)	1443	(1.56)
1984	582	(1.49)	1480	(1.56)
1985	777	(1.29)	1687	(1.57)
1986	488	(1.33)	1546	(1.46)
1987	398	(1.28)	1155	(1.38)
1988	3740	(0.59)	1804	(1.48)
1989	650	(1.12)	4648	(0.85)
1990	374	(1.16)	655	(1.18)
1991	354	(1.10)	516	(1.14)
1992	306	(1.17)	514	(1.13)
1993	390	(1.11)	634	(1.12)
1994	550	(1.00)	919	(1.05)
1995	480	(1.13)	934	(1.05)
1996	831	(0.76)	785	(1.16)
1997	345	(1.00)	1393	(0.90)
1998	419	(0.92)	645	(1.11)
1999	796	(0.82)	1057	(1.02)
2000	438	(1.05)	1231	(1.04)
2001	238	(1.08)	906	(1.14)
2002	205	(1.08)	877	(1.20)
2003	2501	(0.52)	1436	(1.29)
2004	263	(1.12)	7665	(0.78)
2005	274	(1.03)	970	(1.31)
2006	309	(0.94)	1100	(1.14)
2007	179	(1.04)	910	(1.22)
2008	183	(1.07)	1587	(1.04)
2009	301	(1.01)	743	(1.21)
2010	350	(1.22)	823	(1.24)
2011	89484	(0.50)	8760	(1.25)
2012	1319	(1.94)	8624	(1.66)
2013	1506	(1.39)	1978	(1.60)
2014	624	(1.55)	1434	(1.53)
2015	616	(1.54)	1452	(1.56)



**Figure 1:** A summary of the sequence of investigations that led to the conclusion that the younger fish needed to contribute more to the survey biomass index for *S. mentella*, and hence that natural mortality needed to be increased for younger ages (see Introduction section of the main text for further details)

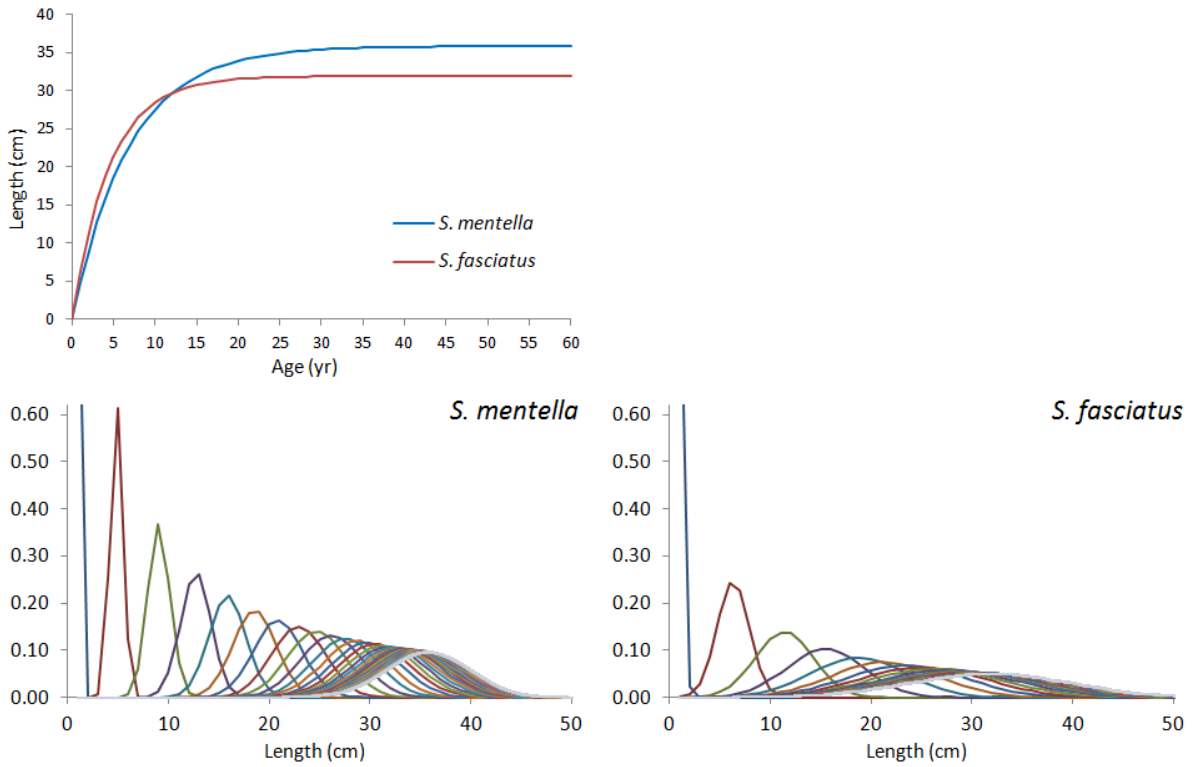


**Figure 2:** A plot of the biomass of the 1981 cohort (ignoring other than natural mortality) which was used to guide the choice of the value of  $M$  of 0.4 for younger ages for *S. mentella*

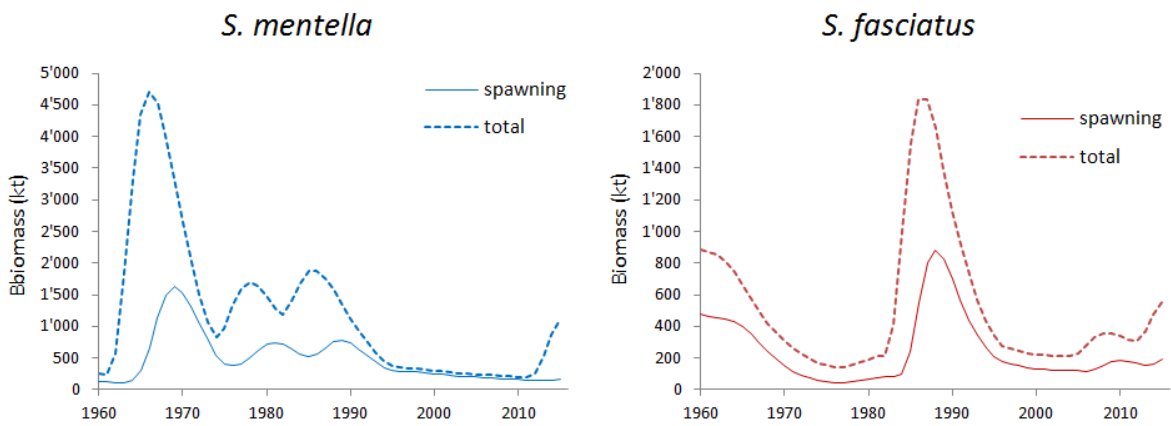


**Figure 3:** The natural mortality vector  $M_a$  used for the *S. mentella* assessment. In the interests of parsimony, this same vector was then adopted for the *S. fasciatus* assessment.

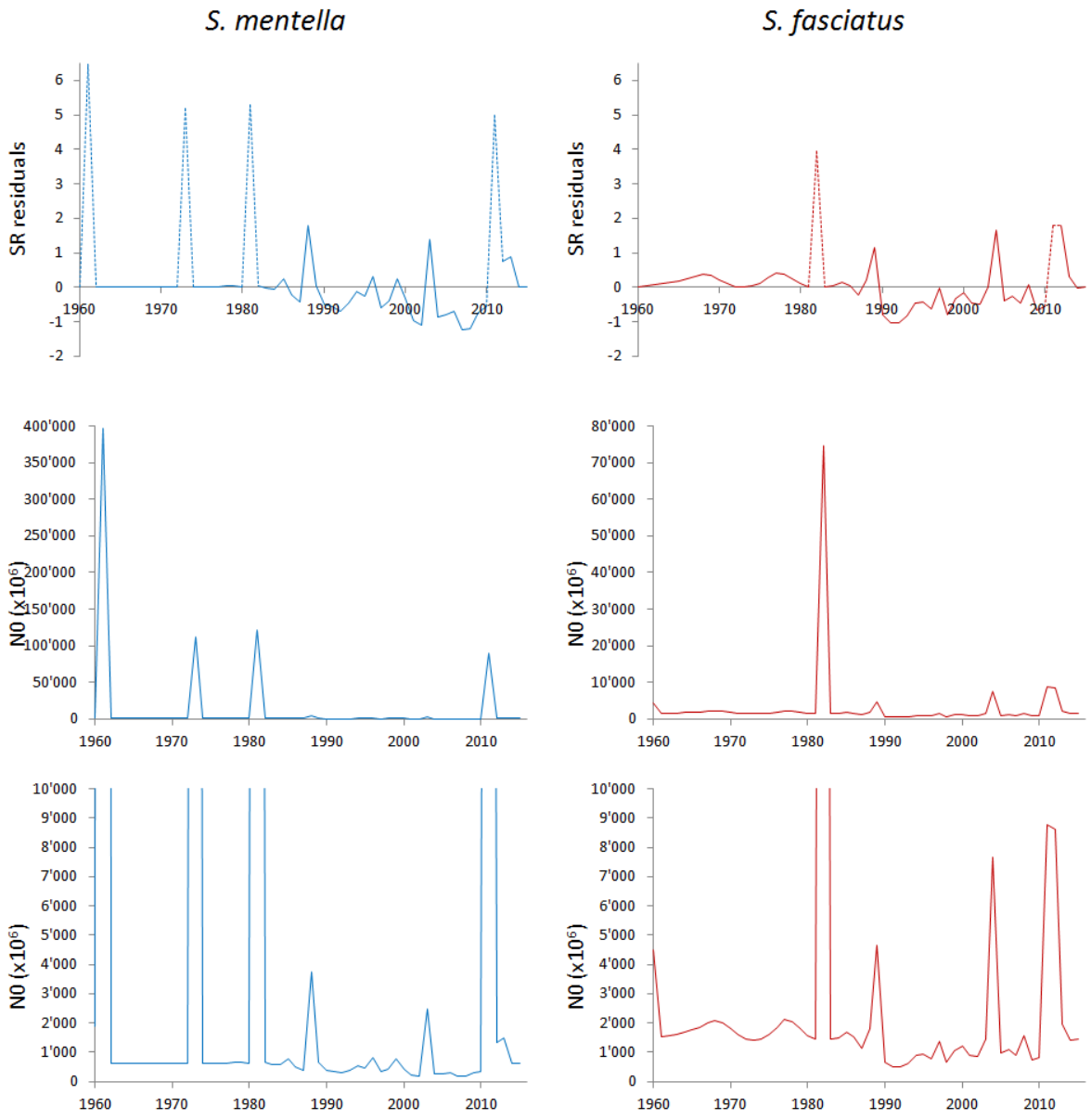




**Figure 4:** Growth curves [Campana, pers. commn] and age-length distributions.



**Figure 5:** Time-series of estimated spawning and total biomass (in kt) for *S. mentella* and *S. fasciatus*. Note that the vertical scales differ.



**Figure 6:** Time-series of estimated stock-recruit residuals and recruitments for *S. mentella* and *S. fasciatus*. The third row has a different vertical scale than the second row to show the lower recruitments better.

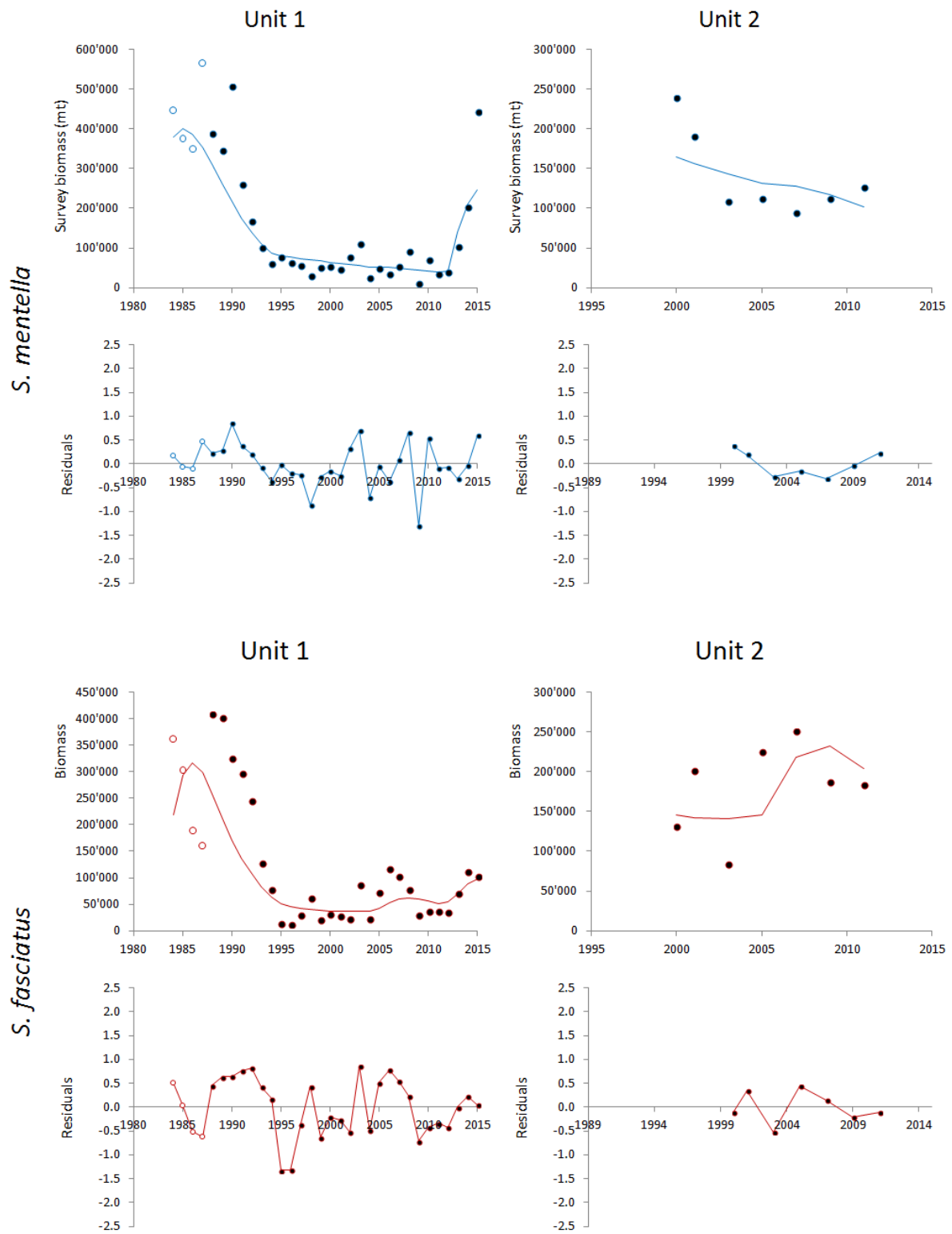
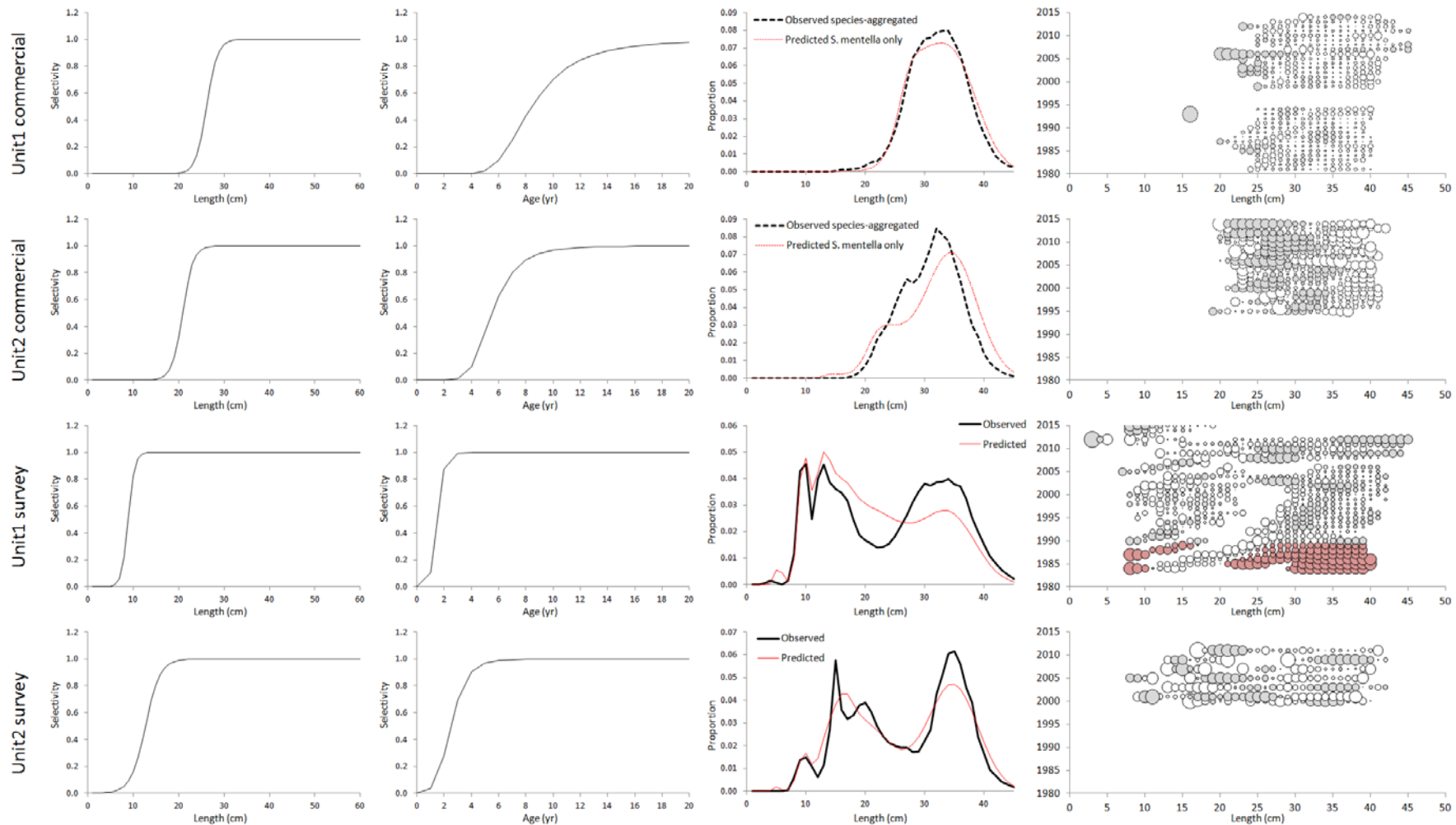
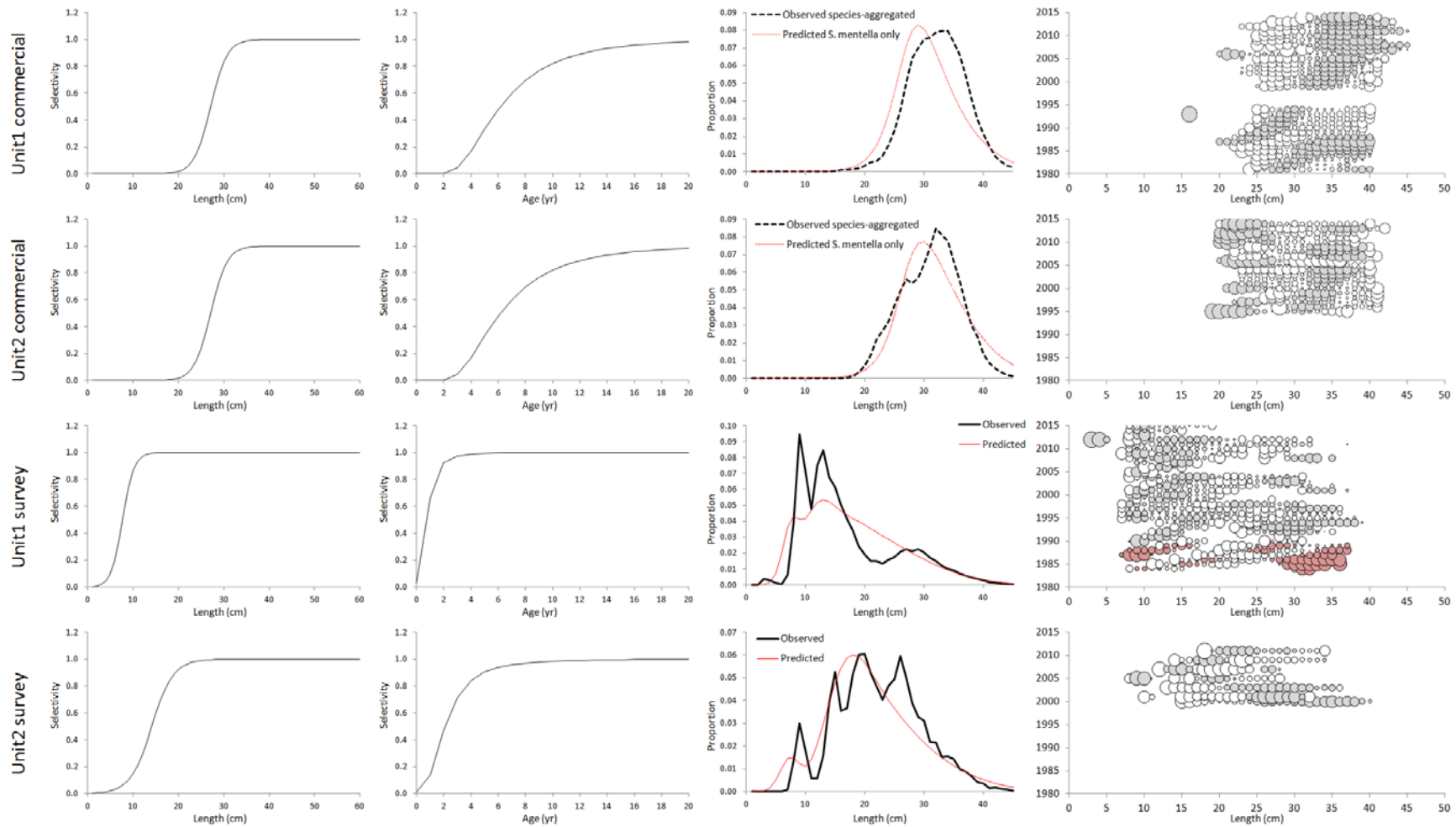


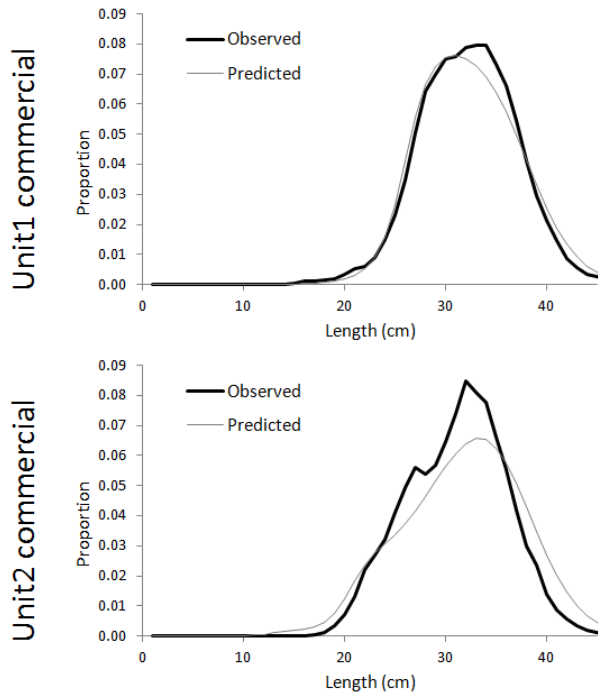
Figure 7: Fits to the survey biomass index data. Open circles represent the Hammond data.



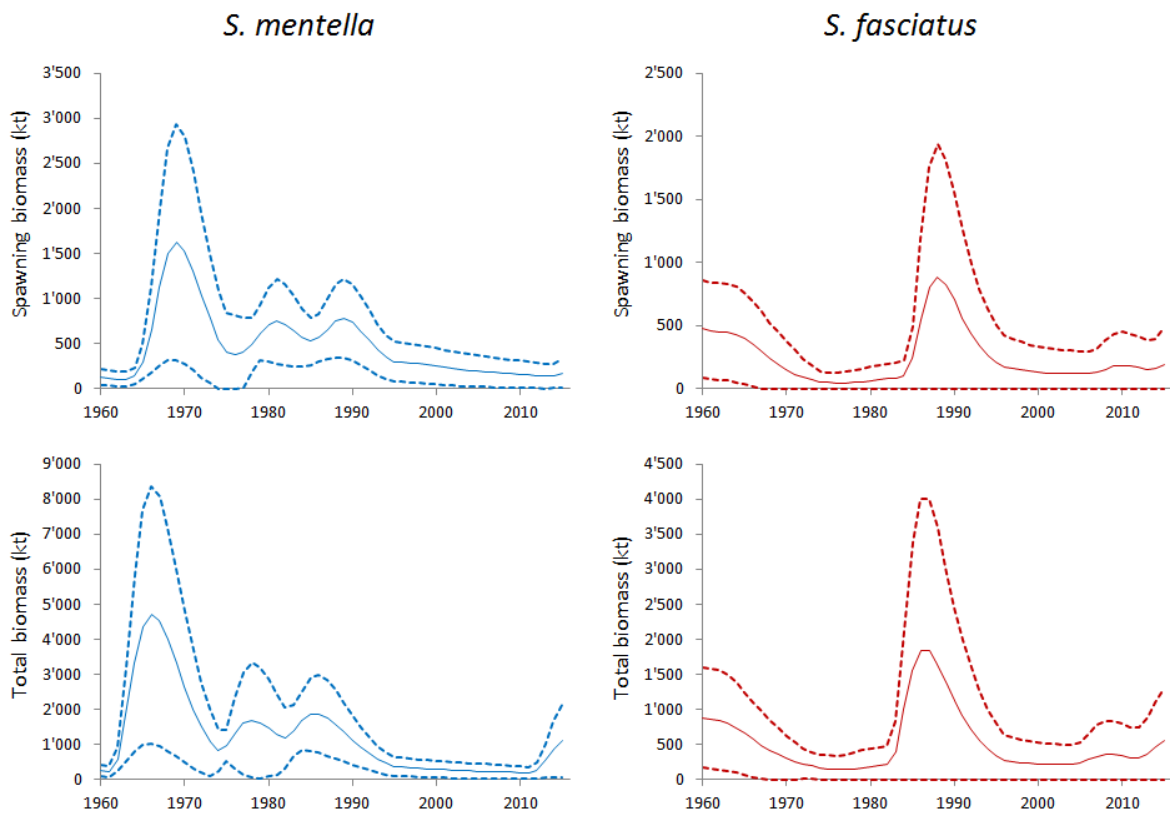
**Figure 8:** *S. mentella*: Estimated selectivity-at-length and -at-age (first and second columns), and fits to the survey catch-at-length data, as averaged over the years for which data are available (third column), and as bubble plots of the standardised residuals (fourth column) (filled bubbles reflect positive residuals, and the bubble area is proportional to the magnitude of the residual). The residuals for the Hammond data (which have been heavily downweighted in their contribution the negative log likelihood) are shown filled in red. For the commercial data, the plots compare the *S. mentella* predicted catch-at-length with the species-aggregated observed catch-at-length; these data are not included in the fit to the model, and the comparisons are shown here simply as a consistency check.



**Figure 9:** *S. fasciatus*: Estimated selectivity-at-length and -at-age (first and second columns), and fits to the survey catch-at-length data, as averaged over the years for which data are available (third column), and as bubble plots of the standardised residuals (fourth column) (filled bubbles reflect positive residuals, and the bubble area is proportional to the magnitude of the residual). residuals for the Hammond data (which have been heavily downweighted in their contribution to the negative log likelihood) are shown filled in red. For the commercial data, the plots compare the *S. fasciatus* predicted catch-at-length with the species-aggregated observed catch-at-length; these data are not included in the fit to the model, and the comparisons are shown here simply as a consistency check.



**Figure 10:** Fits to the species-combined commercial catch-at-length data.



**Figure 11:** Spawning and total biomass trajectories (in kt) with Hessian-based 95% CI (dotted lines) for *S. mentella* and *S. fasciatus*.

## Appendix A - The data

The data have kindly been provided by Daniel Duplisea, pers. commn.

**Table A1:** Catches in mt. The basis for the species splits of these catches, using information from surveys, is set out in McAllister and Duplisea (2012).

	Species combined		<i>S. mentella</i>		<i>S. fasciatus</i>	
	Unit 1	Unit 2	Unit 1	Unit 2	Unit 1	Unit 2
1960	12830	23287	7735	10813	5095	12474
1961	11062	18329	6669	8511	4393	9818
1962	7151	21295	4311	9888	2840	11407
1963	20817	22290	12550	10350	8267	11940
1964	30524	23192	18402	10769	12122	12423
1965	52829	21834	31850	10138	20979	11696
1966	67962	28392	40973	13183	26989	15209
1967	71905	42170	43350	19580	28555	22590
1968	95264	20169	57433	9365	37831	10804
1969	92320	46276	55658	21487	36662	24789
1970	90503	49407	54563	22941	35940	26466
1971	82189	58200	49550	27024	32639	31176
1972	82592	45201	49793	20988	32799	24213
1973	136101	31827	82053	14778	54048	17049
1974	67081	34038	40442	15805	26639	18233
1975	70052	38471	42233	17863	27819	20608
1976	44378	23709	26755	11009	17623	12700
1977	17072	28750	10292	13349	6780	15401
1978	14934	26548	9004	12327	5931	14221
1979	16425	18771	9902	8716	6523	10055
1980	15539	17129	9368	7953	6171	9176
1981	22045	21751	13291	10100	8754	11652
1982	26731	17025	16116	7905	10615	9120
1983	24974	13473	15056	6256	9918	7217
1984	35827	8141	23389	3780	12438	4361
1985	28333	11494	17773	5337	10560	6157
1986	36414	10765	22053	4998	14361	5767
1987	43446	13956	25571	6480	17875	7476
1988	51892	10728	29927	4981	21965	5747
1989	52482	15386	29883	7144	22599	8242
1990	61934	14789	35034	6867	26900	7922
1991	67527	23205	38121	10775	29406	12430
1992	77753	17159	44189	7967	33564	9192
1993	51156	27428	29574	12735	21582	14693
1994	19586	24324	11576	11294	8010	13030
1995	50	12243	31	5685	20	6558
1996	74	9407	46	4368	28	5039
1997	38	9660	25	4485	13	5175
1998	399	10474	272	4863	127	5611
1999	1123	11551	779	5363	343	6188
2000	1192	11553	820	6884	372	4669
2001	1105	9033	740	5169	364	3864
2002	1206	7455	782	4233	423	3222
2003	847	6707	536	3703	311	3004
2004	934	6987	577	3192	357	3795
2005	978	6089	588	2138	390	3951
2006	690	6510	404	2056	286	4454
2007	105	4832	61	1576	45	3256
2008	421	3256	240	1205	180	2051
2009	637	6083	363	2492	275	3591
2010	548	6473	312	2742	236	3731
2011	631	4100	362	1760	269	2340
2012	699	5331	406	2475	293	2856
2013	474	1963	281	912	193	1052
2014	355	2454	216	1139	140	1315
2015	355	2454	222	1139	133	1315





**Table A3:** Survey swept-area total mean biomass for unit 1 and unit 2, species disaggregated.

	Hammond/Needler/ Teleost Unit 1 survey		GEAC Unit 2 survey		Hammond/Needler/ Teleost Unit 1 survey		GEAC Unit 2 survey	
	Swept area mean biomass (mt)	CV	Swept area mean biomass (mt)	CV	Swept area mean biomass (mt)	CV	Swept area mean biomass (mt)	CV
1984	446958	0.11	-		362001	0.27	-	
1985	375753	0.14	-		303334	0.22	-	
1986	348863	0.11	-		189759	0.24	-	
1987	565677	0.39	-		160941	0.19	-	
1988	389407	0.12	-		407910	0.13	-	
1989	346190	0.16	-		400934	0.17	-	
1990	507133	0.33	-		325351	0.10	-	
1991	260826	0.22	-		295856	0.20	-	
1992	170030	0.24	-		244181	0.42	-	
1993	103217	0.52	-		127521	1.21	-	
1994	61208	0.22	-		77270	1.01	-	
1995	79388	0.13	-		13772	0.28	-	
1996	63774	0.21	-		12375	0.28	-	
1997	58211	0.15	-		29836	0.53	-	
1998	29989	0.21	-		62468	1.31	-	
1999	51951	0.29	-		20659	0.34	-	
2000	54655	0.13	238572	0.28	30749	0.19	132154	0.72
2001	47396	0.15	190035	0.22	28271	0.44	201959	1.42
2002	79741	1.87	-		21781	1.07	-	
2003	111024	1.24	108895	0.22	86256	1.03	84063	0.15
2004	26079	0.24	-		22535	0.24	-	
2005	50204	0.12	112695	0.14	71451	0.41	225236	0.31
2006	35822	0.33	-		116550	0.23	-	
2007	54626	0.38	93802	0.23	103010	0.30	251268	0.22
2008	92199	0.85	-		78258	0.38	-	
2009	12257	0.17	111678	0.19	29382	0.21	188343	0.98
2010	72483	1.57	-		37167	0.27	-	
2011	35779	0.19	126273	0.24	37302	0.24	184340	0.36
2012	40188	0.14	-		35770	0.33	-	
2013	103830	0.25	-		69771	0.48	-	
2014	203391	0.14	-		110602	0.22	-	
2015	444778	0.17	-		103134	0.58	-	





**Table A.5:** Life history parameter values assumed for *S. mentella* and *S. fasciatus*. Parameters for the proportion mature-at-length are from the average of the female and male parameters, provided by D. Duplisea, pers. comm. Length-at-age parameters are from Campana, pers. comm.

<b><i>S. mentella</i></b>				
<i>m</i>	60	<i>Maximum age considered (taken to be a plus group)</i>		
<i>M</i>	0.4-0.05	See Figure 3 for details		
<i>h</i>	0.98			
Proportion mature-at-length	<i>p</i>	$\delta$	$mat_l = \frac{\exp(p + \delta l)}{1 + \exp(p + \delta l)}$	
	8.5355	0.3535		
Fraction of <i>M</i> that occurs before spawning ( $M^s$ )	0.25			
	$L_{inf}$	$\kappa$	$t_0$	
Length-at-age	35.81	0.1458	0	$L_a = L_{inf}(1 - e^{-\kappa(a-t_0)})$
	$\alpha$	$\beta$		
Weight-at-length	0.009443	3.107	$(W_l = \alpha l^\beta, l \text{ in cm and } W \text{ in kg})$	
<b><i>S. fasciatus</i></b>				
<i>m</i>	60	<i>Maximum age considered (taken to be a plus group)</i>		
<i>M</i>	0.4-0.05	See Figure 3 for details		
<i>h</i>	0.98			
Proportion mature-at-length	<i>p</i>	$\delta$	$mat_l = \frac{\exp(p + \delta l)}{1 + \exp(p + \delta l)}$	
	10.646	0.493		
Fraction of <i>M</i> that occurs before spawning ( $M^s$ )	0.25			
	$L_{inf}$	$\kappa$	$t_0$	
Length-at-age	31.88	0.2213	0	$L_a = L_{inf}(1 - e^{-\kappa(a-t_0)})$
	$\alpha$	$\beta$		
Weight-at-length	0.01106	3.080	$(W_l = \alpha l^\beta, l \text{ in cm and } W \text{ in kg})$	

## Appendix B - The Statistical Catch-at-Length Model

The text following sets out the equations and other general specifications of the SCAL followed by details of the contributions to the (penalised) log-likelihood function from the different sources of data available and assumptions concerning the stock-recruitment relationship. Quasi-Newton minimization is then applied to minimize the total negative log-likelihood function to estimate parameter values (the package AD Model Builder™ (Fournier *et al.* 2011) is used for this purpose).

### B.1. Population dynamics

#### B.1.1 Numbers-at-age

The resource dynamics are modelled by the following set of population dynamics equations:

$$N_{y+1,1} = R_{y+1} \quad (\text{B1})$$

$$N_{y+1,a+1} = \left( N_{y,a} e^{-M_a/2} - \sum_f C_{y,a}^f \right) e^{-M_a/2} \quad \text{for } 1 \leq a \leq m-2 \quad (\text{B2})$$

$$N_{y+1,m} = \left( N_{y,m-1} e^{-M_{m-1}/2} - \sum_f C_{y,m-1}^f \right) e^{-M_{m-1}/2} + \left( N_{y,m} e^{-M_m/2} - \sum_f C_{y,m}^f \right) e^{-M_m/2} \quad (\text{B3})$$

where

$N_{y,a}$  is the number of fish of age  $a$  at the start of year  $y$  (which refers to a calendar year),

$R_y$  is the recruitment (number of 1-year-old fish) at the start of year  $y$ ,

$M_a$  denotes the natural mortality rate for fish of age  $a$ ,

$C_{y,a}^f$  is the predicted number of fish of age  $a$  caught in year  $y$  by fleet  $f$ , here, the units are considered as fleet and

$m$  is the maximum age considered (taken to be a plus-group).

#### B.1.2. Recruitment

The number of recruits (i.e. new 0-year olds) at the start of year  $y$  is assumed to be related to the spawning stock size (i.e. the biomass of mature fish) at the mid-point of the preceding year by a Beverton-Holt stock-recruitment relationship, allowing for annual fluctuation about the deterministic relationship:

$$R_y = \frac{\alpha B_{y-1}^{\text{SP}}}{\beta + B_{y-1}^{\text{SP}}} e^{(\zeta_y - (\sigma_R)^2/2)} \quad (\text{B4})$$

where

$\alpha$  and  $\beta$  are spawning biomass-recruitment relationship parameters,

$\zeta_y$  reflects fluctuation about the expected recruitment for year  $y$ , which is assumed to be normally distributed with standard deviation  $\sigma_R$  (which is input in the applications considered here); these residuals are treated as estimable parameters in the model fitting process.

$B_y^{\text{SP}}$  is the spawning biomass in year  $y$ , computed as:

$$B_y^{\text{SP}} = \sum_{a=0}^m f_a w_a^{\text{SP}} N_{y,a} e^{-M_a \frac{T^s}{12}} \quad (\text{B5a})$$

with

$$R_0 = K^{sp} \left[ \sum_{a=1}^{m-1} f_a w_a^{sp} e^{-\sum_{a=0}^{a-1} M_a} + f_m w_m^{sp} \frac{e^{-\sum_{a=0}^{m-1} M_a}}{1 - e^{-M_m}} \right] \quad (\text{B5b})$$

where spawning for the stocks under consideration is taken to occur  $T^s$  months after the start of the year (here  $T^s = 3$ ) and some natural mortality has therefore occurred,

$w_a^{sp}$  is the mass of fish of age  $a$  during spawning, and

$f_a = \sum_l f_l A_{a,l}$  is the proportion of fish of age  $a$  that are mature, converted from proportion-at-length, where

$A_{a,l}$  is the proportion of fish of age  $a$  that fall in the length group  $l$  (i.e.,  $\sum_l A_{a,l} = 1$  for all ages).

The matrix  $A_{a,l}$  is calculated under the assumption that length-at-age is normally distributed about a mean given by the von Bertalanffy equation, i.e.:

$$L_a \sim N \left[ L_\infty \left( 1 - e^{-\kappa(a-t_0)} \right), \theta_a^2 \right] \quad (\text{B6})$$

where

$\theta_a$  is the standard deviation of length-at-age  $a$ , which is modelled to be proportional to the expected length-at-age  $a$ , i.e.:

$$\theta_a = \beta L_\infty \left( 1 - e^{-\kappa(a-t_0)} \right) \quad (\text{B7})$$

with  $\beta$  being estimated in the model fitting procedure.

### B.1.3. Total catch and catches-at-age

The total catch by mass in year  $y$  is given by:

$$C_y = \sum_f \sum_{a=0}^m w_{y,a}^f C_{y,a}^f = \sum_f \sum_{a=0}^m w_{y,a}^f N_{y,a} e^{-M_a/2} S_{y,a}^f F_y^f \quad (\text{B8})$$

where

$C_{y,a}^f$  is the catch-at-age, i.e. the number of fish of age  $a$ , caught in year  $y$  by fleet  $f$ ,

$S_{y,a}^f$  is the commercial selectivity of fleet  $f$  (i.e. combination of availability and vulnerability to fishing gear) at age  $a$  for year  $y$ ; when  $S_{y,a} = 1$ , the age-class  $a$  is said to be fully selected,

$F_y^f$  is the proportion of a fully selected age class that is fished by fleet  $f$ , and

$w_{y,a}^f$  denotes the selectivity-weighted mid-year weight of fish of age  $a$  landed in year  $y$  by fleet  $f$ , computed as:

$$\tilde{w}_{y,a}^f = \sum_l S_{y,l}^f w_l A_{a,l} / S_{y,a}^f \quad (\text{B9})$$

with

$w_l$  is the weight of fish of length  $l$ ; and

Selectivity is estimated as a function of length and then converted to an effective selectivity-at-age:

$$S_{y,a}^f = \sum_l S_{y,l}^f A_{a,l} \quad (\text{B10})$$

### B.1.4. Initial conditions

For the first year ( $y_0$ ) considered in the model (here 1960), the numbers-at-age are taken to be at unexploited equilibrium, i.e.:

$$N_{y_0,a} = \begin{cases} R_0 & \text{for } a = 0 \\ N_{y_0,a-1} e^{-M_{a-1}} & \text{for } 1 \leq a \leq m-1 \\ N_{y_0,a-1} e^{-M_{a-1}} / (1 - e^{-M_m}) & \text{for } a = m \end{cases} \quad (\text{B11})$$

## B.2. The (penalised) likelihood function

The model is fitted to survey biomass indices and commercial and survey catch-at-length data to estimate model parameters (which may include residuals about the stock-recruitment function, facilitated through the incorporation of a penalty function described below). Contributions by each of these to the negative of the (penalised) log-likelihood ( $-\ell nL$ ) are as follows.

### B.2.1 Survey biomass indices

The likelihood is calculated assuming that the survey index observed for a particular unit is log-normally distributed about its expected value:

$$I_y^i = \hat{I}_y^i \exp(\varepsilon_y^i) \quad \text{or} \quad \varepsilon_y^i = \ln(I_y^i) - \ln(\hat{I}_y^i) \quad (\text{B12})$$

where

$I_y^i$  is the survey biomass index for year  $y$  for survey series  $i$ ,

$\hat{I}_y^i = \hat{q}^i \sum_{a=1}^m w_{y,a}^i S_a^i N_{y,a} e^{-Z_{y,a} \frac{T^i}{12}}$  is the corresponding model estimate of biomass,

$\hat{q}^i$  is the constant of proportionality (catchability) for the survey series,

$T^i$  is the timing (month) of survey series  $i$ , and

$\varepsilon_y^i$  from  $N(0, (\sigma^i)^2)$ .

The contribution of the survey data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\ell nL^{\text{surv}} = \sum_y \left\{ \ln \left( \sqrt{(\sigma^i)^2 + (\sigma_{Add}^i)^2} \right) + \frac{(\varepsilon_y^i)^2}{2[(\sigma^i)^2 + (\sigma_{Add}^i)^2]} \right\} \quad (\text{B13})$$

where

$\sigma^i = \sqrt{1/n^i \sum_y (\ln(I_y^i) - \ln(\hat{I}_y^i))^2}$  is the standard deviation of the residuals for the logarithm of index  $i$ , and

$\sigma_{Add}^i$  is the square root of the additional variance for the survey series, which is estimated in the model fitting procedure with the constraint  $\sigma_{Add}^i \geq 0.1$ .

The catchability coefficient  $q^i$  for survey index  $i$  is estimated by its maximum likelihood value:

$$\ell n \hat{q}^i = 1/n_i \sum_y (\ln I_y^i - \ln \hat{B}_y^i) \quad (\text{B14})$$

with

$B_y^i = u^i \sum_a^m w_{y,a}^i S_{y,a}^i N_{y,a} e^{-Z_{y,a} \frac{T^i}{12}}$ , the estimate of biomass available to survey  $i$ ; and

$u^i$  the proportion of the total biomass available to survey  $i$  (which is input, see Table 1).

### B.2.3. Survey and commercial catches-at-length

The contribution of the catch-at-length data to the negative of the log-likelihood function under the assumption of an “adjusted” lognormal error distribution (Punt and Kennedy 1997) is given by:

$$-\ell n L^{\text{CAL}} = w_{\text{len}}^i \sum_f \sum_y \sum_l \left[ \ell n \left( \sigma_{\text{len}}^i / \sqrt{p_{y,l}^i} \right) + p_{y,l}^i \left( \ell n p_{y,l}^i - \ell n \hat{p}_{y,l}^i \right)^2 / 2 \left( \sigma_{\text{len}}^i \right)^2 \right] \quad (\text{B15})$$

where

$p_{y,l}^i = C_{y,l}^i / \sum_{l'} C_{y,l'}^i$  is the observed proportion of fish caught in year  $y$  by fleet/survey  $i$  that are of length  $l$ ,

$\hat{p}_{y,l}^i = \hat{C}_{y,l}^i / \sum_{l'} \hat{C}_{y,l'}^i$  is the model-predicted proportion of fish caught in year  $y$  by fleet/survey  $i$  that are of length  $l$ ,

where

$$\hat{C}_{y,l}^i = \sum_a N_{y,a} A_{a,l} S_{y,l}^i e^{-Z_{y,a} \frac{T^i}{2}} \quad (\text{B16})$$

and

$\sigma_{\text{len}}^i$  is the standard deviation associated with the catch-at-length data of fleet/survey  $i$ , which is estimated in the fitting procedure by:

$$\hat{\sigma}_{\text{len}}^i = \sqrt{\sum_y \sum_l p_{y,l}^i \left( \ell n p_{y,l}^i - \ell n \hat{p}_{y,l}^i \right)^2 / \sum_y \sum_l 1} \quad (\text{B17})$$

Catches-at-length proportions are aggregated so that the minimum proportion is 1%.

The  $w_{\text{len}}^i$  weighting factor may be set to a value less than 1 to downweight the contribution of the catch-at-length data (which tend to be positively correlated between adjacent length groups) to the overall negative log-likelihood compared to that of the survey biomass data. Here  $w_{\text{len}}^i = 0.1$  for the Needler/Teleost survey catch-at-length data. The contribution of the Hammond catch-at-length data to the negative log-likelihood is further downweighted by an additional multiplier of 1/20 for reasons explained in the text.

Since the commercial catch-at-length data are species-aggregated, it was not possible to use them directly in the likelihood for fitting the species-disaggregated assessments considered here.

### B.2.4. Stock-recruitment function residuals

The stock-recruitment residuals are assumed to be log-normally distributed. Thus, the contribution of the recruitment residuals to the negative of the (now penalised) log-likelihood function is given by:

$$-\ell n L^{\text{SR}} = \sum_{y=y_1+1}^{y_2} \left[ \zeta_y^2 / 2\sigma_{\text{R}}^2 \right] \quad (\text{B18})$$

where

$\zeta_y$  is the recruitment residual for year  $y$ , which is estimated for year  $y_1$  to  $y_2$  (see equation (B4)),



$\sigma_R$  is the standard deviation of the log-residuals, which is input (here  $\sigma_R = 1.5$ ).

Certain years with peak (extraordinarily strong) recruitments are not included in the summation in equation B18. For *S. mentella*, the years are 1961, 1973, 1981, and 2011. For *S. fasciatus*, the years are 1982 and 2011. The recruitments for these years are treated as unconstrained estimable parameters.

#### **B.2.5. Penalty on the survey catchability coefficients**

$$- \ell nL^q = \sum_i \left[ 2 \frac{(q^i - lb)}{(ub - lb)} - 1 \right]^{16} \quad (B19)$$

where

$lb$  and  $ub$  are the lower and upper bounds imposed, here 0.1 and 2 respectively.

#### **B.2.6. Catch penalty**

A penalty is included so that the predicted catches correspond to those observed:

$$- \ell nL^{\text{catch}} = \sum_i \left[ \frac{(\ln C_y^i - \ln \hat{C}_y^i)^2}{2\sigma_C^2} \right] \quad (B20)$$

where

$\sigma_C$  is the standard deviation of the catches, which is input (here  $\sigma_C = 0.2$ ).

#### **B.3. Fishing selectivity**

Fishing selectivities-at-length are estimated using a logistic form:

$$S_l = \left( 1 + e^{(b-l)/a} \right)^{-1} \quad (B21)$$

#### **B.4. Estimation of precision**

Where quoted, 95% probability interval estimates are based on the Hessian.

## Annex to : Statistical Catch-At-Length assessment results for *Sebastes mentella* and *S. fasciatus* in Units 1 and 2

R A Rademeyer and D S Butterworth

This document contains the following further results:

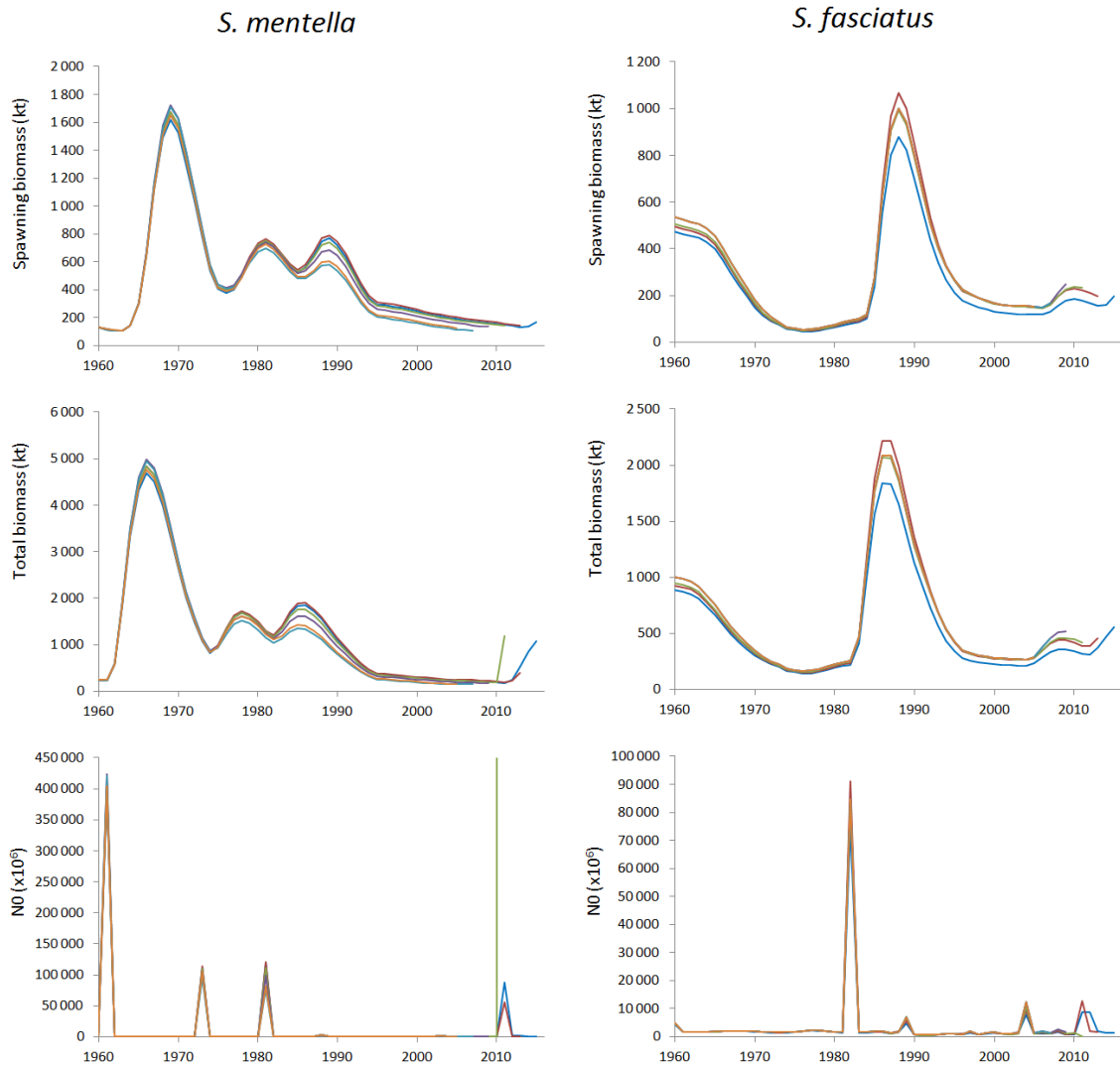
Figure A.1: Retrospective analyses for *S. mentella* and *S. fasciatus*.

Table A.1 and Figures A.2 and A.3: Results for a run with a smoother *M*-at-age for *S. mentella* and *S. fasciatus* respectively.

Figures A.4 and A.5: Fit to the survey CAL data for each year for *S. mentella* and *S. fasciatus* respectively.

**Table A.1:** Negative log-likelihood contributions for the RC and Smoother  $M$  runs for *S. mentella* and *S. fasciatus*.

		<i>S. mentella</i>		<i>S. fasciatus</i>	
		RC	Smoother M	RC	Smoother M
Total $-\ln L$		46.8	46.6	68.4	70.5
$-\ln L^{\text{catch}}$		-76.2	-76.3	-77.1	-77.0
$-\ln L^{\text{survey}}$	unit 1	21.1	22.4	29.1	30.1
	unit 2	1.3	1.2	2.4	2.5
caa_nll	unit 1 surv	26.6	24.9	32.6	32.3
	unit 2 surv	1.4	1.4	6.6	6.7
$-\ln L^{\text{sr}}$		72.6	72.9	74.9	75.9
$-\ln L^{\text{q}}$		0.0	0.0	0.0	0.0



**Figure A.1:** Restropective analyses for *S. mentella* and *S. fasciatus* (10 years, every 2 years).

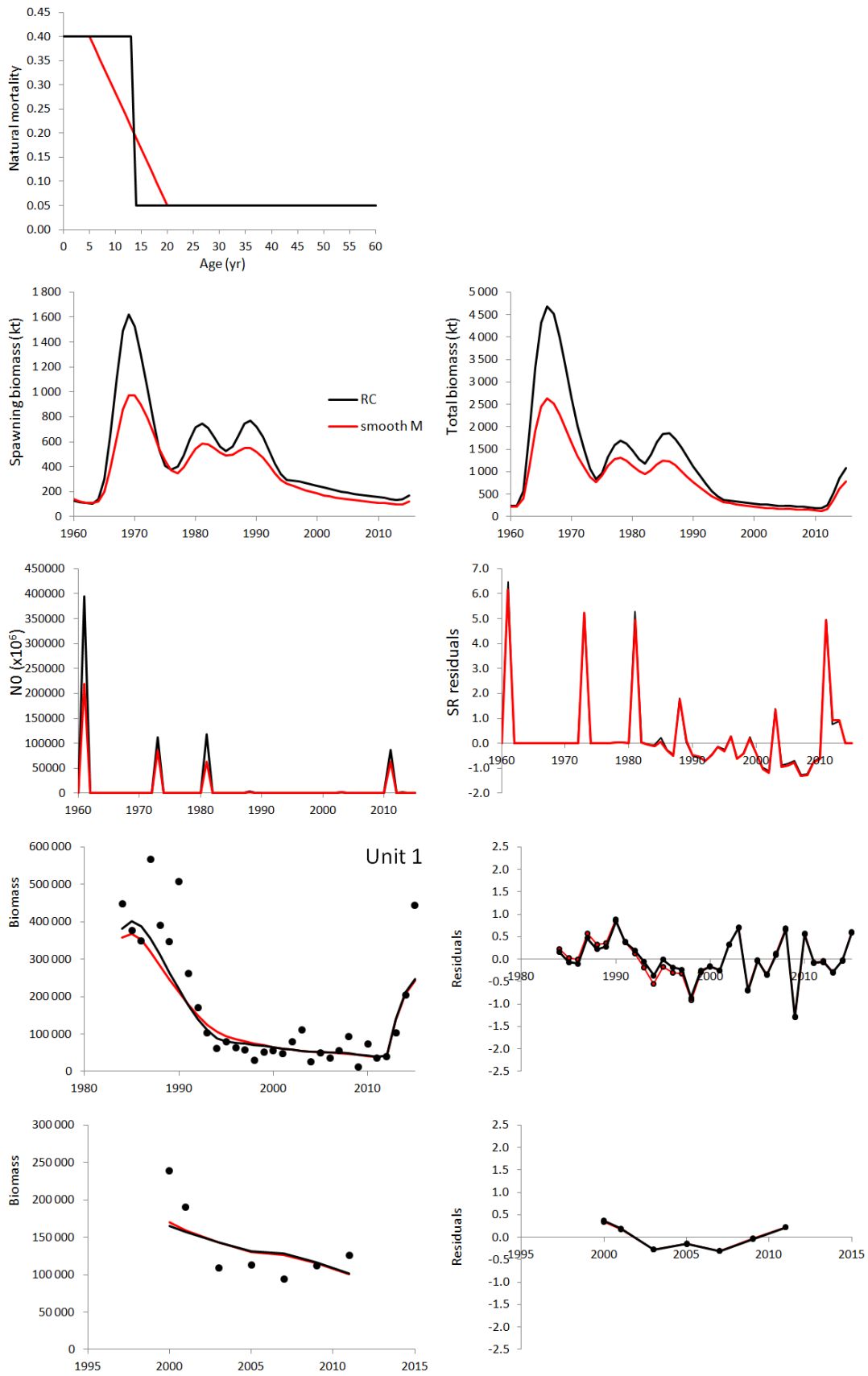
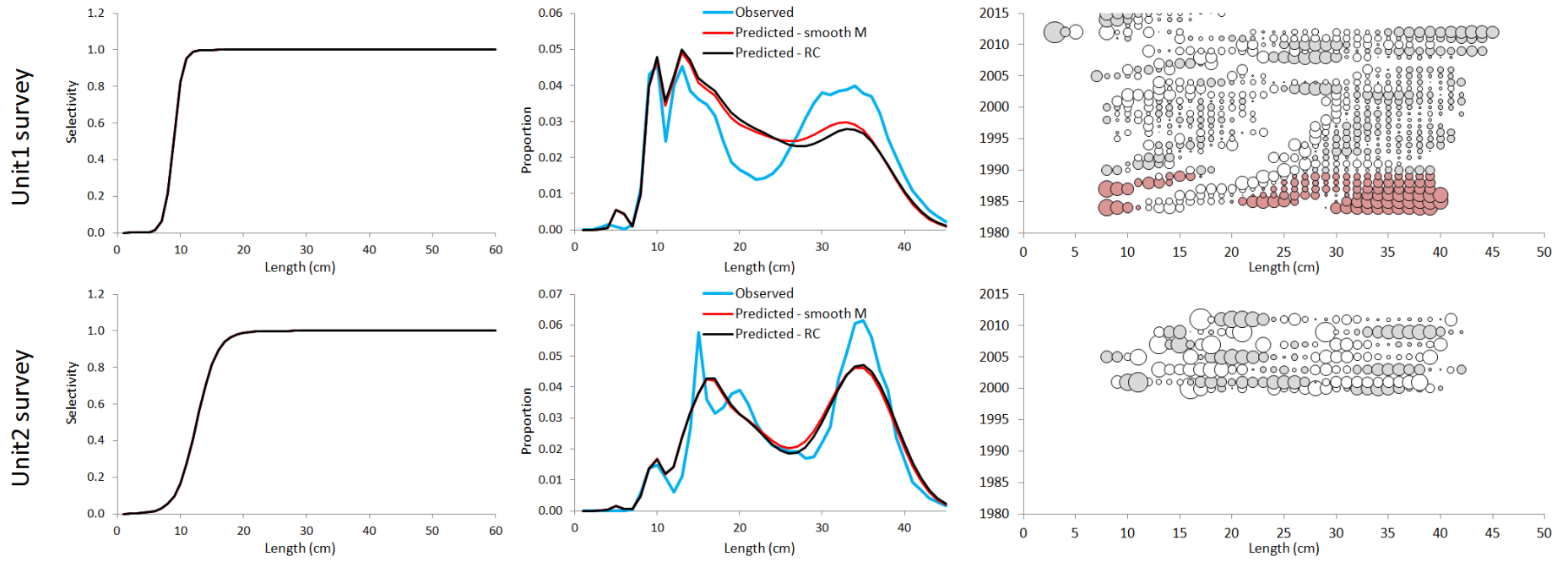


Figure A.2a: *S. mentella* results for the smooth  $M$  option compared to the RC.



**Figure A.2b:** *S. mentella* results for the smooth  $M$  option compared to the RC. Note: the bubble plots are for the smooth  $M$  option only.

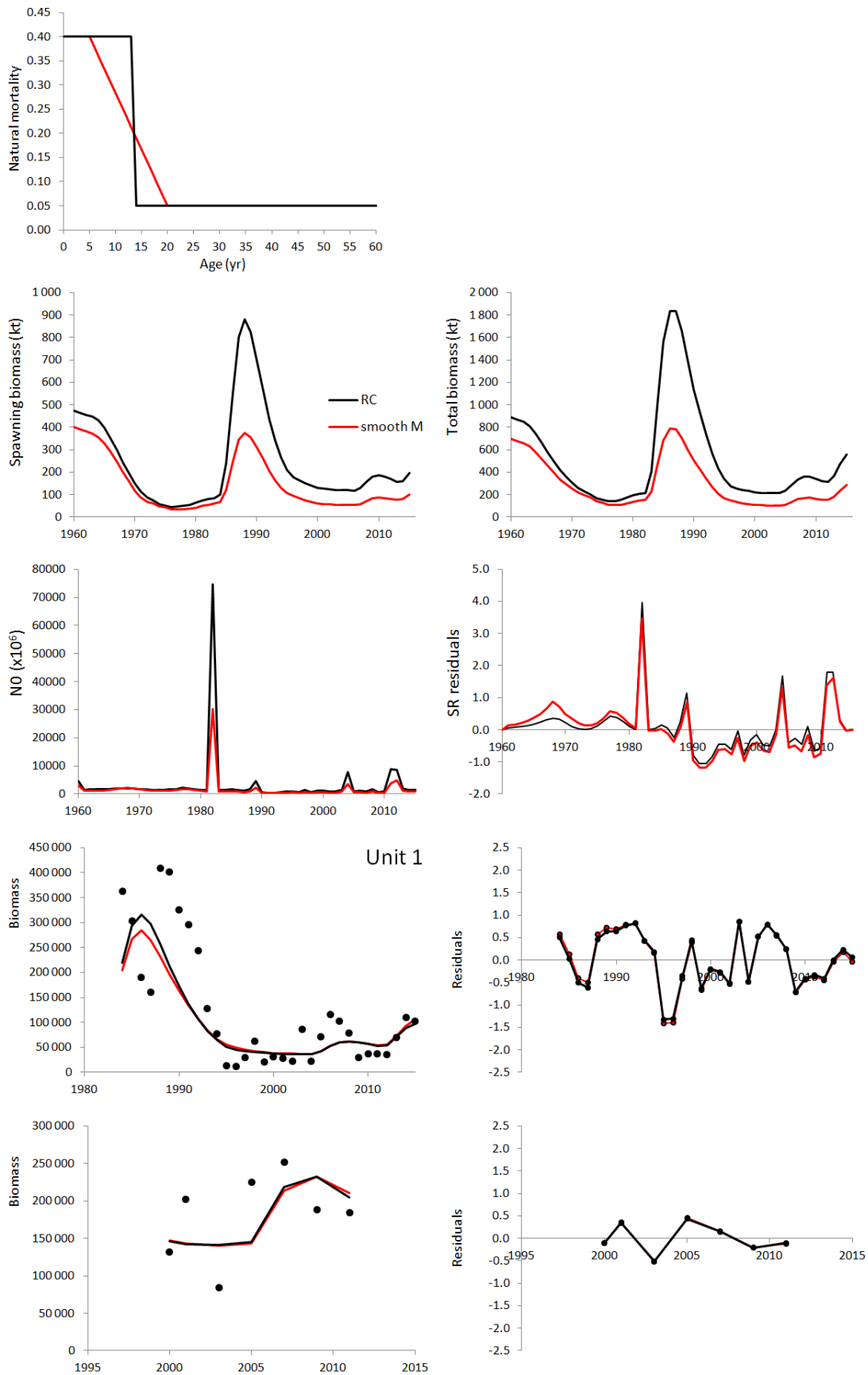
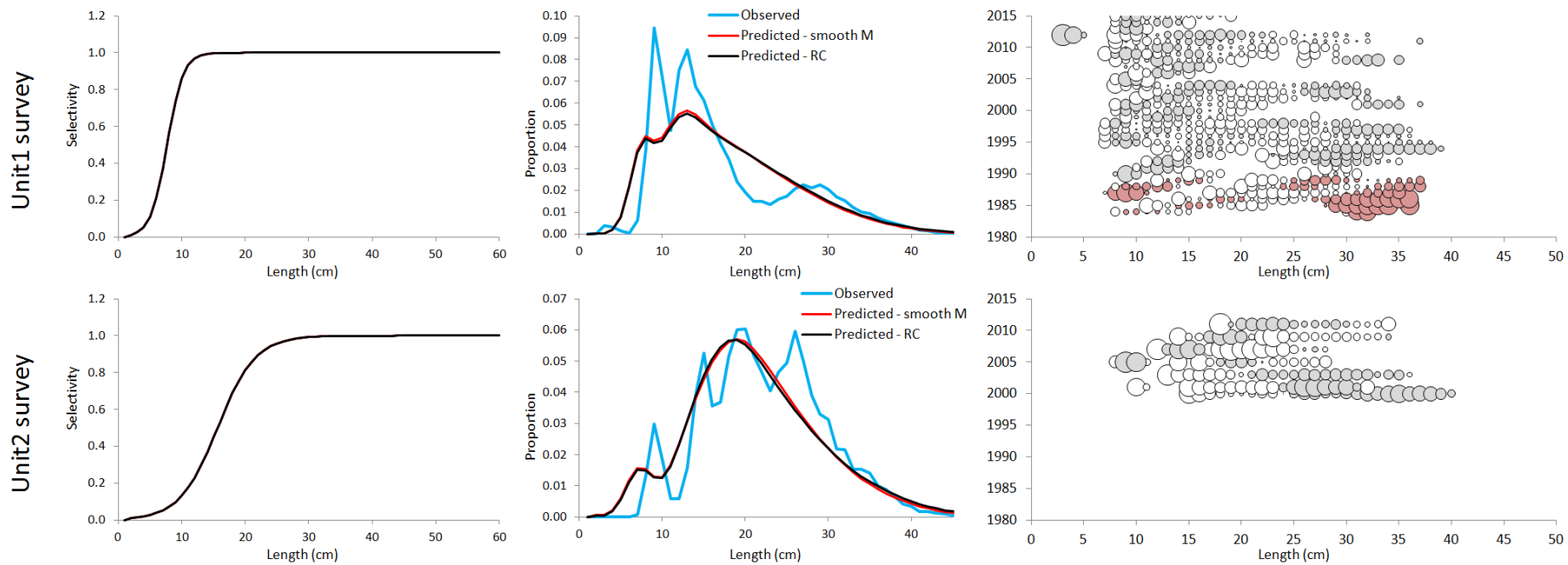
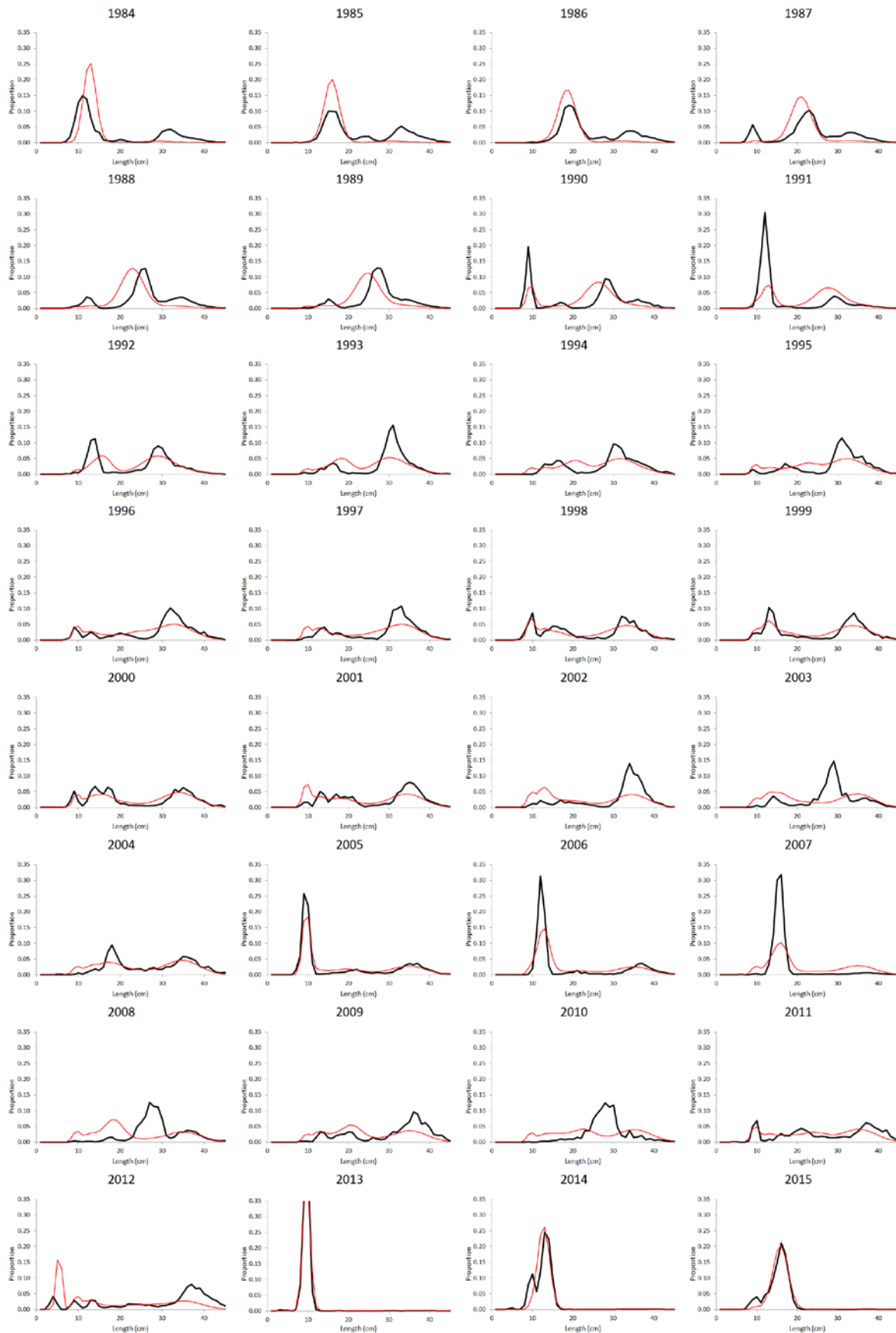


Figure A.3a: *S. fasciatus* results for the smooth  $M$  option compared to the RC.

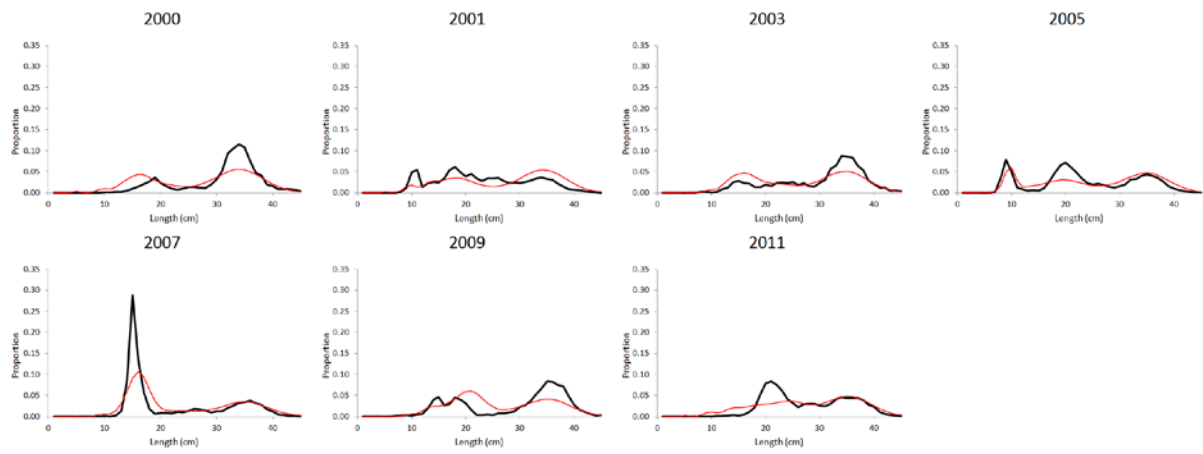


**Figure A.3b:** *S. fasciatus* results for the smooth  $M$  option compared to the RC. Note: the bubble plots are for the smooth  $M$  option only.

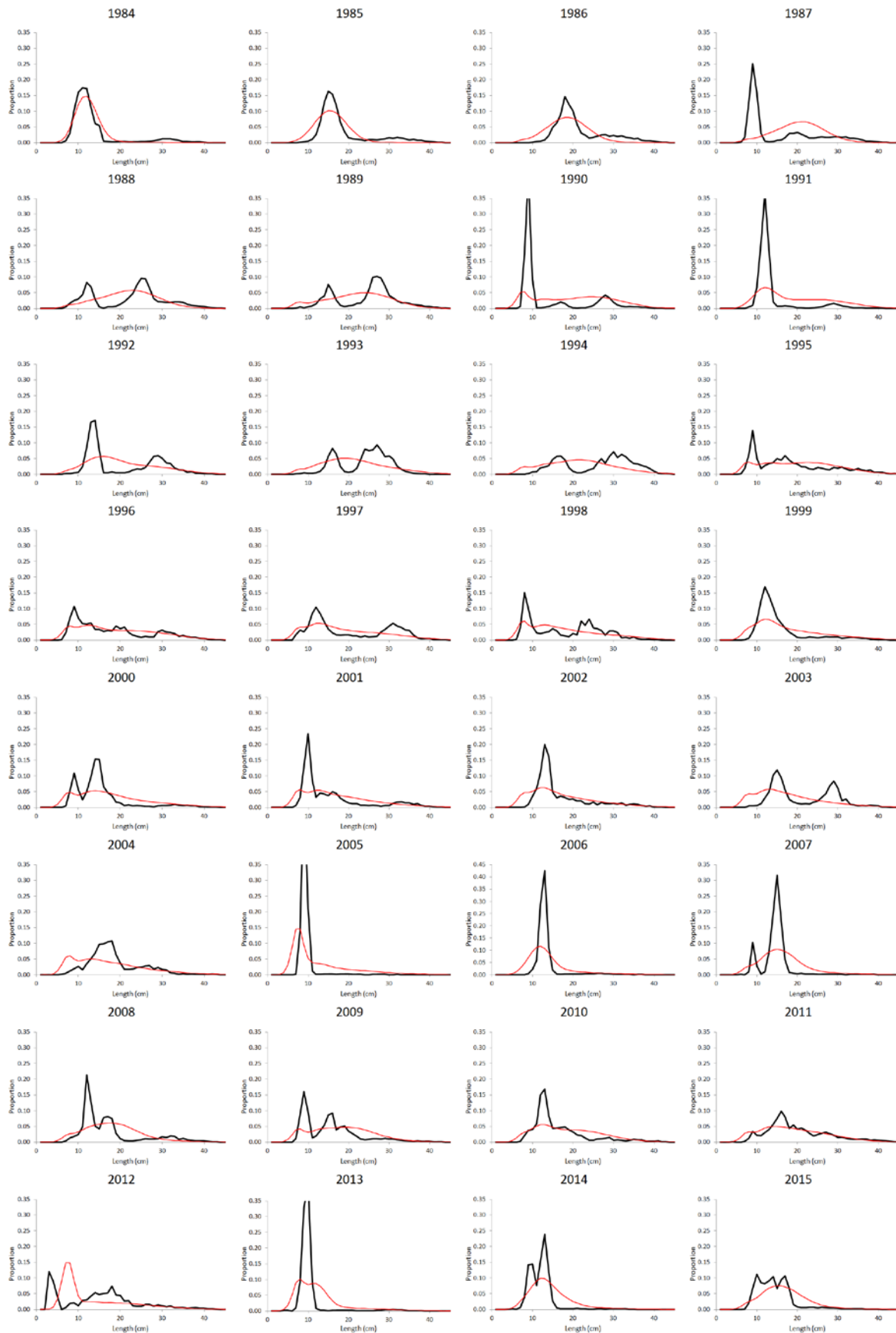




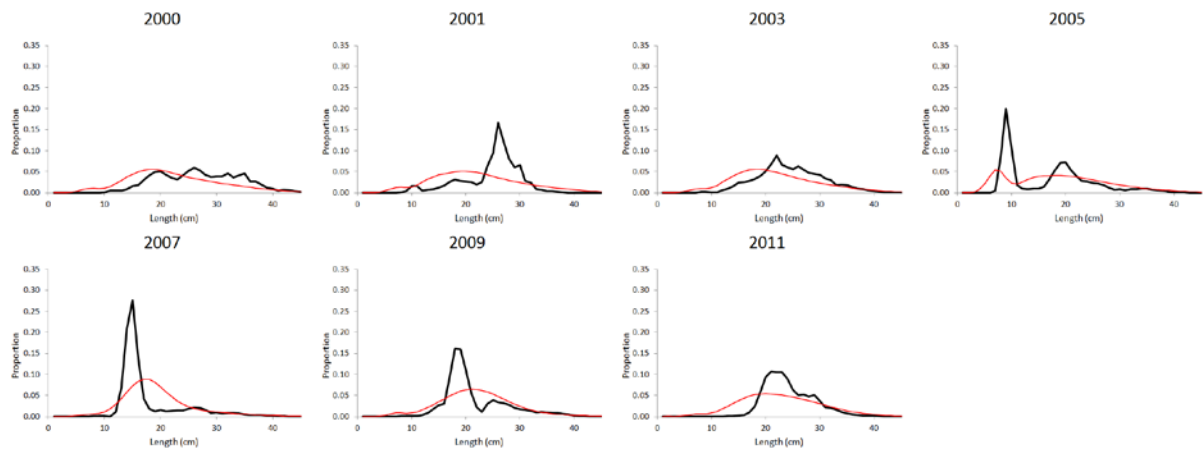
**Figure A.4a:** Fit to the Unit 1 survey CAL data for *S. mentella*. The observed CAL are in black while the model predicted CAL are in red.



**Figure A.4b:** Fit to the Unit 2 survey CAL data for *S. mentella*. The observed CAL are in black while the model predicted CAL are in red.



**Figure A.5a:** Fit to the Unit 1 survey CAL data for *S. fasciatus*. The observed CAL are in black while the model predicted CAL are in red.



**Figure A.5b:** Fit to the Unit 2 survey CAL data for *S. fasciatus*. The observed CAL are in black while the model predicted CAL are in red.